Chapter 1: Intro. To Machinery Principles

Electrical Machines

Generator: Mechanical energy ⇨ Electrical energy

Motor: Electrical energy ⇨ Mechanical energy

We will study the following machines:

- **Synchronous generator and motor**
- **Induction motor**
- **DC motor**

We will also look into **transformers** – useful in electrical power distribution.

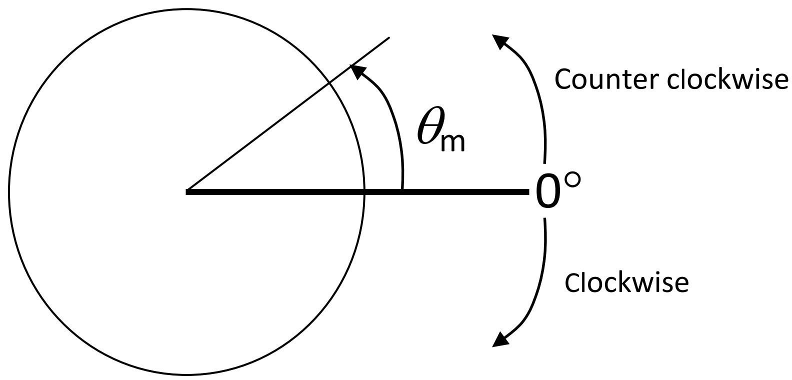
Firstly, we need to look at the **basic concepts** of electrical machines:

- Rotational motion and Newton's Law of rotation
- Magnetic field and magnetic circuits
- Principles behind motor, generator and transformer action
- The Linear DC machine

1.1. Rotational Motion

Machines rotate on a fixed shaft.

θ_m = **Angular position** measured from an arbitrary reference point.



Unit: radians [rad] or degrees [°]

ω_m **= Angular velocity**

It is analogous to linear velocity, *v*. Angular velocity is defined as the rate of change in angular position w.r.t. time. Therefore,

$$\omega_m = \frac{d\theta_m}{dt}$$

Unit: radians per second [rads^{-1}].

Angular velocity can also be expressed in terms of other units.
f_m = **Angular velocity** in **revolutions per second**.

n_m=**Angular velocity** in **revolutions per minute.**

The relationship of ω_m , f_m , and n_m with each other is as shown,

$$n_m = 60 f_m$$

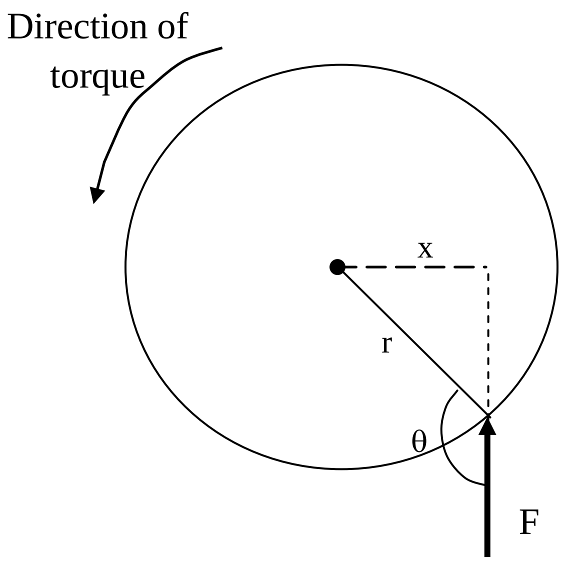
$$f_m = \frac{\omega_m}{2\pi}$$

α = Angular acceleration

It is analogous to linear acceleration, a. It is the rate of change in angular velocity w.r.t. to time. Hence,

Unit: radians per second squared (rads^{-2}).

Torque, *T* or $\boldsymbol{\tau}$ is produced when a force exerts a twisting action on a body. When an object is rotating, its angular velocity changes in the presence of a torque.



$$\begin{aligned}\tau &= \text{(force applied)(perpendicular distance)}\\ &= (F)(r\sin\theta)\\ &= rF\sin\theta \quad \text{[Nm]}\end{aligned}$$

1.1.1 Newton's Law of Rotation

For an object moving in a straight line, Newton's Law is given by:

$$F = ma$$

where

F = net force applied to the object
m = mass of object
a = resulting acceleration of object

In analogy, **Newton's Law of rotation** for a rotating body is given by:

$$\tau = J\alpha$$

where

τ = net torque applied to the object [Nm]
J = moment of inertia of the object [kgm^2]
α = resulting angular acceleration of object [$rads^{-2}$]

Work, *W* is produced from the application of force through a distance i.e. F through a distance, r.

For linear motion:

$$W = \int F\,dr$$
$$= Fr$$

for force that is collinear with the direction of motion.

For a rotational motion, work = **application of torque *T* through an angle θ**

$$W = \int \tau\ d\theta$$
$$= \tau\theta$$

For constant torque applied.

Unit: Joules (J).

Power, P is the rate of doing work or increase in work per unit time.

$$P = \frac{dW}{dt}$$

Unit: Watts (W).

Applying this definition to rotating bodies, and assuming torque is constant,

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\omega$$

The equation for power of a rotating body is **very important**!
It **describes the mechanical power on the shaft** of a motor or generator.

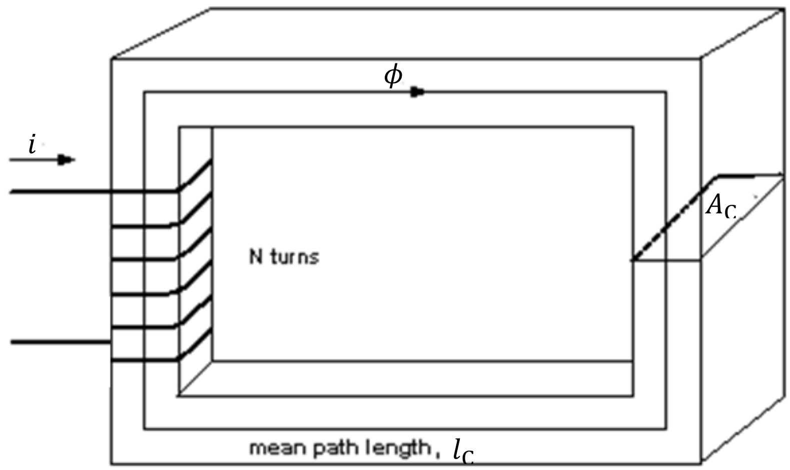
1.2. The Magnetic Field

The conversion of energy from one form to the other in motors, generators and transformers is through the presence of the magnetic field.

The production of a magnetic field by a current carrying conductor is governed by **Ampere's Law**:

$$\oint \overline{H} \cdot dl = I$$

$\overline{H}$ is **the magnetic field intensity** produced by the current *I*. In SI units, $\overline{H}$ is measured in **Ampere-turns per meter** and *dl* is the differential length along the path of integration.



Consider a rectangular iron/ferromagnetic material with a winding of N turns wrapped around one leg.

Then, use the total current passing through the closed path of mean path length l_C is,

$$Hl_C = Ni$$

Therefore, the magnitude of the magnetic field intensity in the core due to the applied current is, $H = \frac{Ni}{l_C}$.

The magnetic field intensity $\overline{H}$ can be considered to be a measure of **the "effort" required** by the current **to create** a **magnetic field**.

The relationship between the **magnetic field intensity, *H*** and the **produced magnetic flux density, *B*** is given by:

$$\bar{B} = \mu\bar{H} \quad [\mathrm{T}]$$

where μ is the **permeability of the material** in which the magnetic field is produced. It represents the **relative ease of establishing a magnetic field** in a given material.

The permeability is usually written as: $\mu = \mu_r\mu_0$
where:

μ_0= permeability of free space = 4π x 10^{-7} H/m
μ_r = relative permeability of a given material compared to free space.

Note:

- Relative permeability of air = permeability of free space = 1
- Steel used in modern machines have μ_r of 2000 to 6000.

The magnitude of the flux density in the core is given as,

$$B = \mu H = \mu \frac{Ni}{l_C}$$

Finally, we define the **magnetic flux** present in a given area by the following equation:

$$\phi = \int_{A_C} \bar{B} \cdot d\bar{A}$$

where $d\bar{A}$ is the differential unit of area. If the flux density, *B* is uniform over the cross-sectional area *A* and is perpendicular to the plane of area A, then:

$$\phi = BA \;\; \text{[Wb]}$$

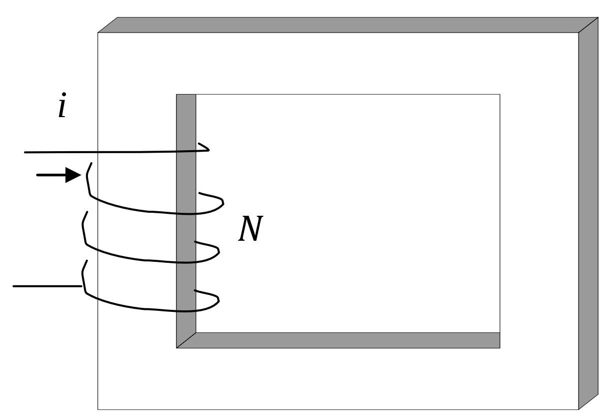
The total flux in the core due to the current *i* in the winding is,

$$\boldsymbol{\phi = BA = \frac{\mu N i A_C}{l_C}}$$

1.2.1 Magnetic Circuits

It is possible to define a **magnetic circuit** in which magnetic behaviour is governed by **simple equations analogous to** those of an **electric circuit**.

Therefore, for the simple magnetic core:



The **magnetomotive force,** F is **equal to the effective current flow applied** to the core, i.e. $\underline{F = Ni}$

Electric Circuit Analogy	Magnetic Circuit
A voltage in an electric circuit produces ➢ **Current flow**	Current in a coil of wire wrapped around a core produces ➢ **Magnetic flux**
The electromotive force is given as $\boldsymbol{V = IR}$ [**V**]	The magnetomotive force is as $F = \phi\mathcal{R}$ [A · turns]
I = current [A]	ϕ = flux [Webers]
R = resistance of circuit [Ω]	$\mathcal{R}$ = reluctance of circuit [A · turns/Webers]

The magnetomotive force in a magnetic circuit also has a polarity associated to it.

Positive mmf is at the end which the **flux exits**.
Negative mmf is at the end which the **flux enters**.

This is **determined by the flux flow** in the magnetic circuit determined using the '*right-hand rule*':

"If fingers of the right hand curl in the direction of the current flowing in a coil of wire, the thumb will point in the direction of positive mmf."

Due to the analogy, reluctances in a magnetic circuit obey the same rules as resistances, i.e.

1. If the reluctances are **in series**, the equivalent reluctance is,

2. if the reluctances are **in parallel**, the equivalent reluctance is,

In order to obtain an expression for the reluctance, we look back at the flux expression for the simple magnetic core obtained previously:

$$\phi = BA = \frac{\mu N i A_C}{l_C}$$

$$\phi = Ni\frac{\mu A_C}{l_C}$$

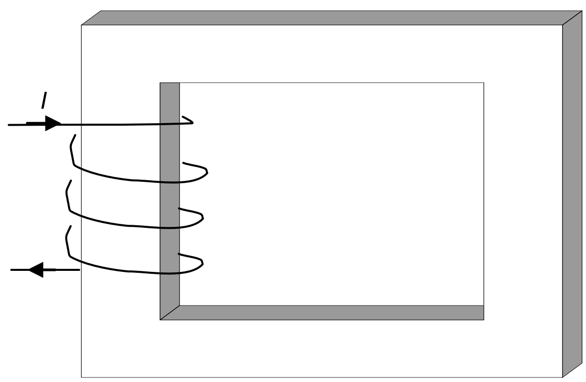
We know earlier that $F = \phi\boldsymbol{\mathcal{R}}$

Hence, the reluctance of a material of length l and cross-sectional area A is given by:

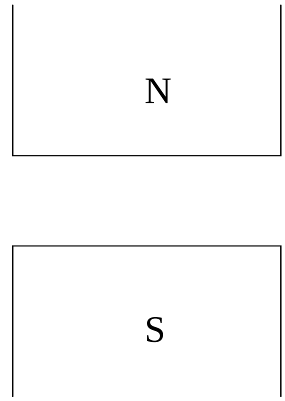
Permeance, $\mathcal{P}$ = reciprocal of reluctance (analogous to conductance G)

Magnetic circuits assist in analysing magnetic problems. However, the analysis carried out is based on **approximations** by assuming the following:

1. **reluctance calculations** (mean path length and cross-sectional area)
2. no **leakage flux** (all flux confine within a magnetic core**)**
3. no **fringing effects** – cross-sectional area of air gap equals that of core.
4. **Permeability** of **ferromagnetic materials** is usually assumed to be **constant.**



Leakage fluxes present in a simple magnetic core.



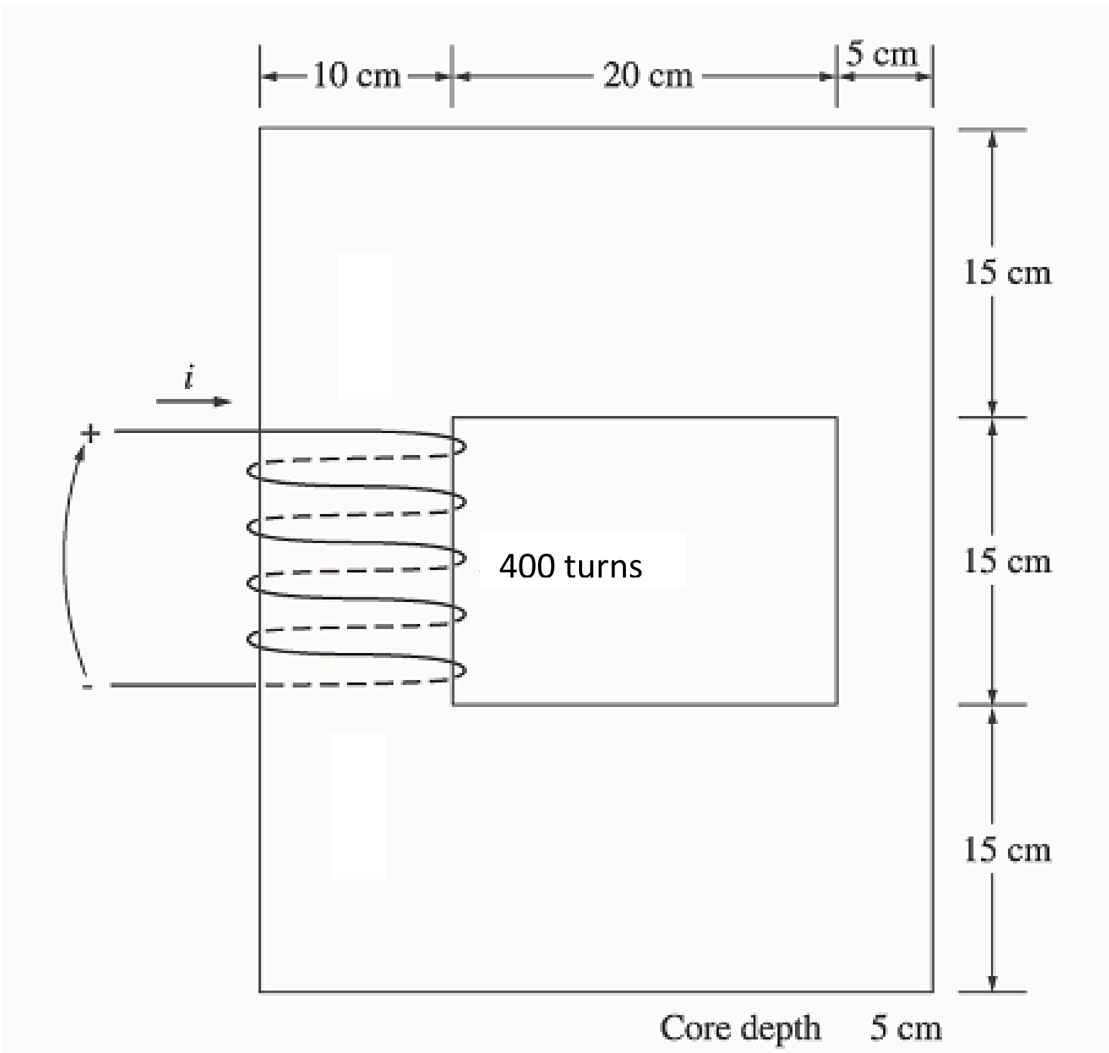
Fringing effects in airgap.

Hence, effective cross-sectional area of airgap is **larger** than cross-sectional

Even so, magnetic circuit analysis is the **easiest tool** for flux calculations giving satisfactory results.

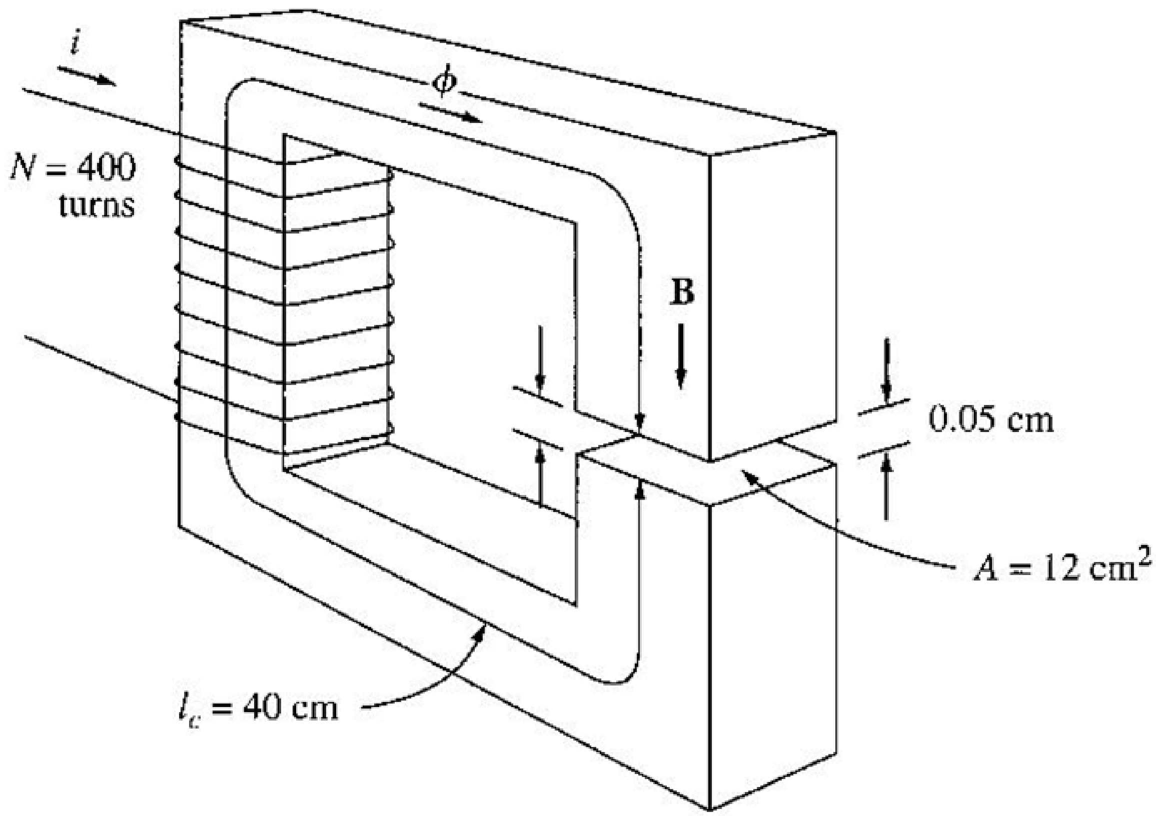
Example 1.1

A ferromagnetic core is shown in Figure P1-2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.005 Wb. With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 1000.



Example1.2

Figure shows a ferromagnetic core whose mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The csa of the core is 12cm2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective csa of the gap by 5%. Given this information, find
(a) the **total reluctance** of the flux path (iron plus air gap)
(b) the **current** required to produce a flux density of 0.5T in the air



Example 1.3

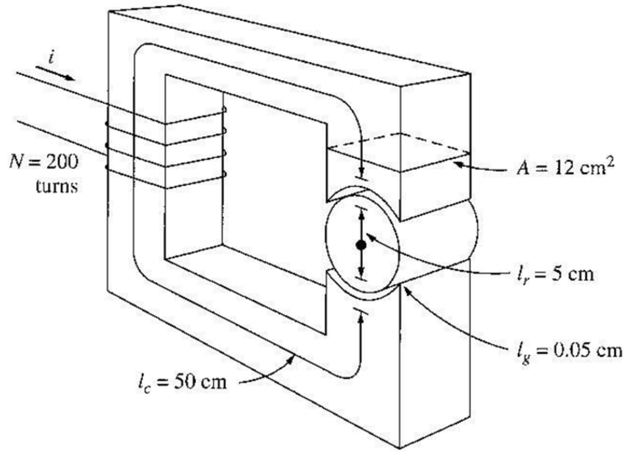
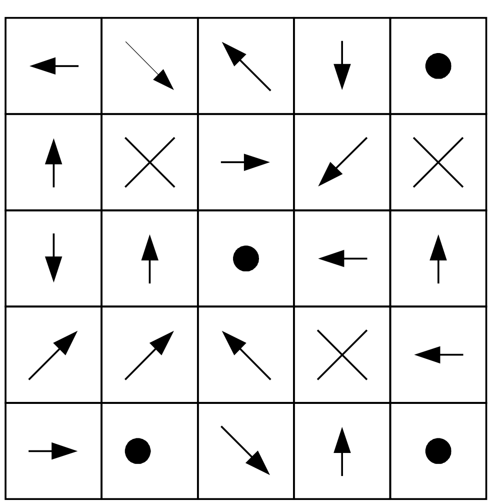


Figure shows a simplified rotor and stator for a dc motor. The mean path length of the stator is 50cm, and its csa is 12cm2. The mean path length of the rotor is 5 cm, and its csa also may be assumed to be 12cm2. Each air gap between the rotor and the stator is 0.05cm wide, and the csa of each air gap (including fringing) is 14cm2. The iron of the core has a relative permeability of 2000, and there are 200 turns of wire on the core. If the current in the wire is adjusted to be 1A, what will the **resulting flux density in the air gaps** be?

1.3. Magnetic behaviour of ferromagnetic materials

The stator and rotor cores of ac and dc machines are made of ferromagnetic materials.

Before magnetic field is applied:



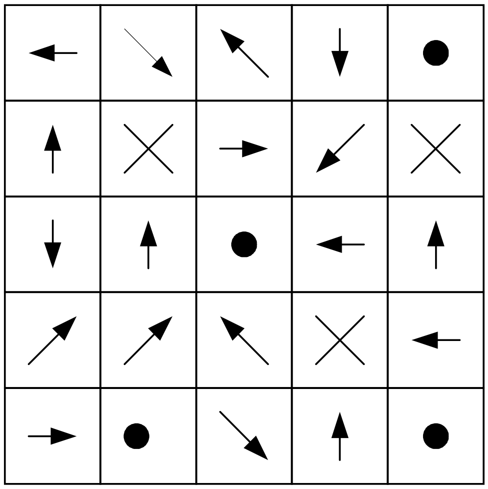
Within a metal, there are small regions ⇒

In **each domain**, atoms are aligned with a small magnetic field.

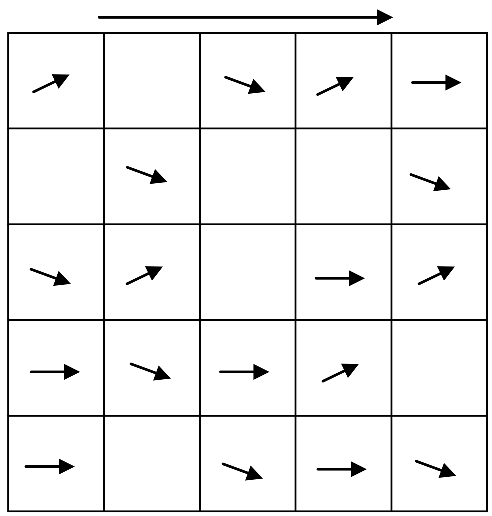
But each **domain field** are **randomly aligned** in material.

Example of metal structure before the presence of a magnetic field.

<u>When external magnetic field is applied:</u>



Before external field applied



Domains pointing **in direction** of field **grow**

Domains in **other directions realign** to **follow external field.**
Hence, **magnetic field increases!**

As more domains align, the total magnetic flux will maintain at a constant level, i.e. any increase in magnetomotive force will not cause much increase in magnetic flux. Iron is saturated with flux.

<u>When magnetic field is removed:</u>

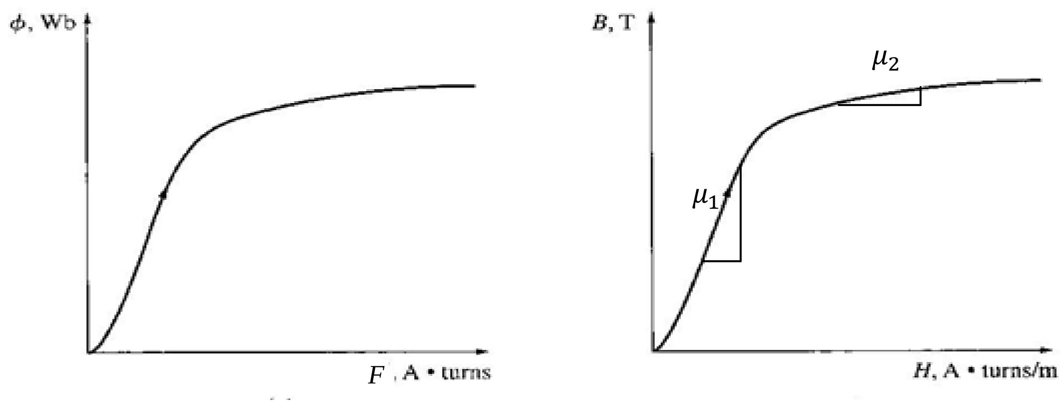
Domains will try to revert to its random state.

But **some remain aligned.** The piece of iron is now a permanent magnet.

Hence, **to change alignment such that net field = 0**, must apply energy!

1. Apply mmf in the opposite direction
2. Exert a large mechanical shock
3. Heat up the material

So, turning domains in a ferromagnetic structure requires energy.

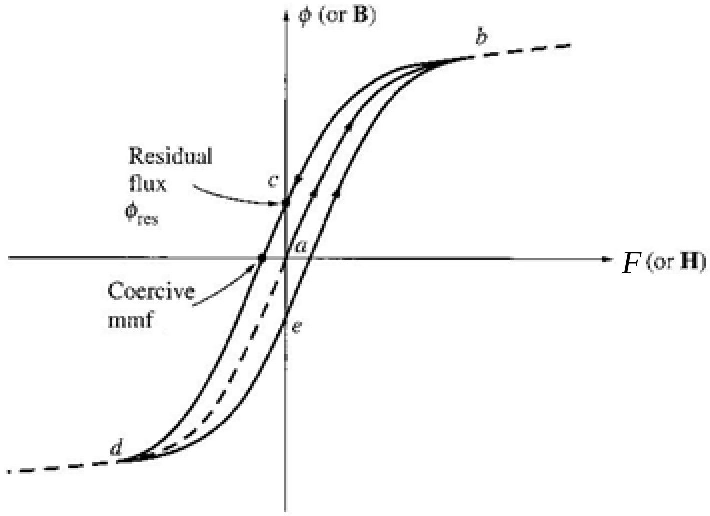


Slope of B-H curve = permeability, μ
Clearly, $\mu \neq$ **constant** in ferromagnetic materials.

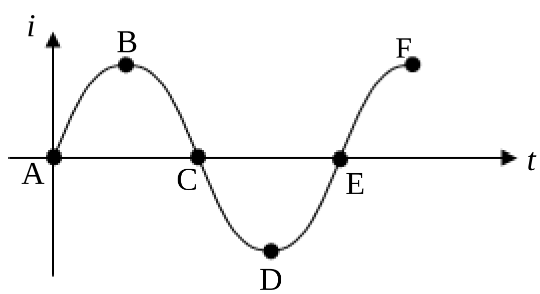
After a certain point, increase in mmf gives almost no increase in flux, i.e. material has **saturated**.

"Knee" of curve – transition region, **operation point** for most electrical machines.

Advantage: get **higher B for a given value of H**. Since generators and motors depend on magnetic flux to produce voltage and torque, designed to produce as much flux as possible.



What will happen when current changes direction (i.e. have alternating current)?



A (a) ⇒ assume **flux** in core is **zero** at $t = 0$

A–B ⇒ current increases, **flux increases** as well (as in saturation curve seen previously).

B–C–D⇒ Current decreases but flux traces <u>**different path**</u>

D–E–F⇒ Current increases again but flux path doesn't go through *a* as seen before.

When **MMF** is **applied and removed**, the flux **path *abc*** is traced.

At c, *F* = 0 but flux ≠ 0 ⇒ Residual flux (precisely the manner permanent magnet is produced)

To force flux = zero (i.e. *B* = 0) $\Rightarrow$ an amount of mmf known as the **coercive force** must be applied

This phenomenon is known as **Hysteresis**.

Energy losses in ferromagnetic core

Two types of losses:

1. ***Hysteresis loss*** – **energy** required to accomplish the **reorientation of domains** during each cycle of **ac current** applied to the core.

   Trajectory of flux built-up in material is different for increasing and decreasing current applied, i.e. hysteresis loop.

   Every cycle of AC current will drive the material around the hysteresis loop once.

   Energy loss $\propto$ area enclosed in hysteresis loop.

2. ***Eddy current loss*** – produced by **induced currents** in the material.
3.

Both losses **cause heating of core material** and needs to be considered in machine or transformer design.

Since both occur within the metal core, these losses are lumped together and called **core losses**.

FACT:

1. Current-carrying wire produces a magnetic field, *B*.
2. Existence of ferromagnetic material (mainly iron) increases *B* and provides easy path for magnetic flux flow.

Electrical machines (motors or generators) and transformers are devices made up of iron and windings carrying current.

The **basic principles** behind the operation of these devices are **caused by the effect of magnetic field on its surroundings**:

- Effect 1: Presence of a coil of wire in a time-changing magnetic field induces voltage (**transformer** action)
- Effect 2: Force is induced on a current-carrying wire in the presence of magnetic field (**motor** action)
- Effect 3: A moving wire in presence of a static magnetic field induces voltage (**generator** action)

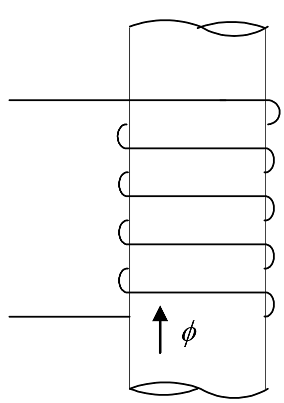
1.4. Effect 1: Faraday's Law

"Flux ϕ passing through a turn of coil induces voltage e_{ind} in it that is proportional to the rate of change of flux with respect to time."

Faraday's Law in equation form: $e_{\text{ind}} = -\frac{d\phi}{dt}$

or for a coil having N turns: $e_{\text{ind}} = -N\frac{d\phi}{dt}$

Negative sign is an expression of **Lenz's Law**. It states that the direction of voltage build up in the coil is such that if the coil ends were short circuited, it would produce a current that would cause a flux opposing the original flux change.



Minus sign can be left out since the polarity of the resulting voltage can be determined physically.

In practical problem the **flux present** in each of N turns is not exactly the same due to leakages. When the flux leakages is considered,

Rewrite Faraday's Law: $\boldsymbol{e_{\text{ind}} = \frac{d\lambda}{dt}}$
where λ **is** flux linkage **of the coil with *N* turns** given by

Units: Weber-turns.

Faraday's Law is the basis of transformer action, i.e. have static coils (or conductors) in a varying magnetic field.

But Faraday's Law also applies if you have:
- Moving conductor in a stationary field
- Moving conductor in a varying field

Back to ***eddy current losses***…

Cause: The time-varying flux **induces voltage** around the core in the same manner in a wire wrapped around the core. The voltage produces current known as *eddy current* within the core.

Effect: **heat is dissipated due to the eddy current** flowing within the resistive core. **Energy loss ∝ size of current paths.**

Solution: **Lamination of ferromagnetic core**, i.e. break up core into thin strips, **separated by insulation** to limit the areas in which eddy currents can flow.

1.5. Effect 2: Induced force on a current carrying wire

Charges moving in a magnetic field experience a force.
If the moving charges are a current flowing in a conductor, a force acting on the conductor is observed.

General equation for the force induced on the conductor:

$$\bar{F} = i(\bar{l} \times \bar{B})$$

where i is the magnitude of current in the conductor

$\bar{l}$ is length of wire; with its direction defined to be the direction of the current flow

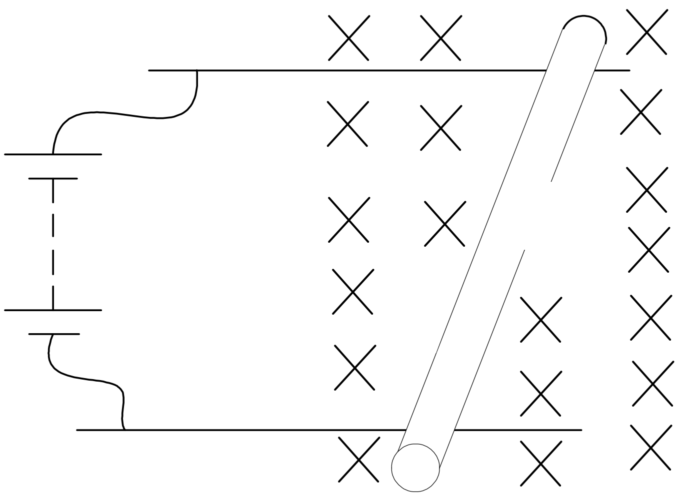
$\bar{B}$ is the magnetic flux density vector

Hence, **force magnitude:**

$$F = ilB \sin\theta$$

(θ = angle between conductor and the flux density vector)

Example: A conductor placed on rails connected to a DC voltage source in a constant magnetic field.



Since all vectors are perpendicular:

$$F = ilB$$

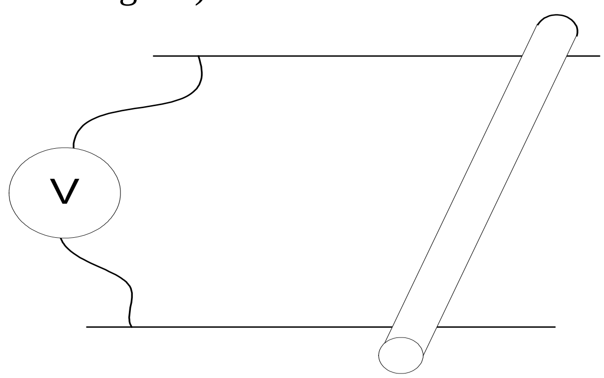
In summary, this phenomenon is the basis of an **electric motor** where torque or rotational force of the motor is the effect of the stator field current and the magnetic field of the rotor.

In electrical motors, construction is such that the windings (i.e. current) and magnetic field are all acting in perpendicular directions.

Why? To achieve maximum force!

1.6. Effect 3: Induced voltage on a moving wire

Now, take the same conductor on rails example above. But, take of the DC voltage source and connect and voltmeter instead. (Note: The conductor is still placed in a constant magnetic field region.)



Then, move the conductor to the right.

Voltage is induced in the system!

General equation for the induced voltage:

$$e_{\text{ind}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$$

where $\bar{v}$ is the velocity of the wire
$\bar{B}$ is the magnetic flux density vector
$\bar{l}$ is the length of the conductor in the magnetic field. It points along the direction of the wire towards the making the smallest angel w.r.t. the vector $\bar{v} \times \bar{B}$

Note: The value of *l* is dependent upon the angle at which the wire cuts through the magnetic field. Hence a more complete formula will be as follows:

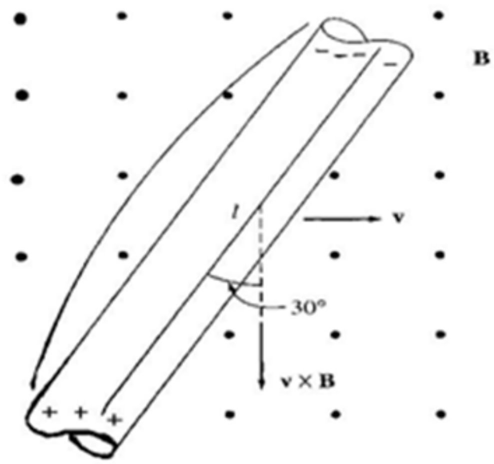
$$e_{\text{ind}} = vB \sin\theta_{\text{vb}}\, l \cos\theta$$

where θ = angle between the conductor and the direction of the $\left(\vec{v} \times \vec{B}\right)$ vector.

This effect is **basis of generator action**, i.e. induction of voltages in a moving wire located in a magnetic field.

Example

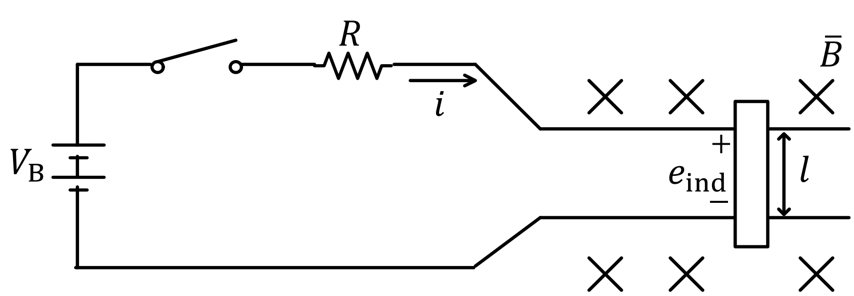
Figure shows a conductor moving with a velocity of 10m/s to the right in a magnetic field. The flux density is 0.5T, out of the page, and the wire is 1m in length. What are the magnitude and polarity of the resulting induced voltage?



1.7. The Linear DC machine

It operates on the same principles and exhibits the same behaviours as real generators and motors.

Construction: A battery is connected through a switch to a conducting bar placed on a pair of smooth, frictionless rails in a constant, uniform magnetic field.



To investigate its behaviour, 4 basic equations are required:

1. Force production on a wire in the presence of a magnetic field:

$$\bar{F} = i(\bar{l} \times \bar{B})$$

2. Voltage induced on a wire moving in a magnetic field:

$$e_{\text{ind}} = (\bar{v} \times \bar{B}) \cdot l$$

3. Kirchhoff's voltage law for the machine:

$$V_{\text{B}} = iR + e_{\text{ind}}$$

4. Newton's law for the bar lying across the rails:

$$F_{\text{net}} = ma$$

The fundamental behaviour of the simple DC machine will be examined through three cases.

Case 1: Starting the Linear DC machine

1. The switch is closed and current is allowed to flow in the bar. From Kirchhoff's voltage law:

Note: $e_{\text{ind}} = 0$ because the bar is at rest.

2. With current flowing downwards in the bar, **force is produced** on it.

$$F_{\text{ind}} = ilB$$

Direction of movement: **to the right**

3. Based on Newton's law, bar will accelerate to the right. When the velocity of the bar increases, a **voltage is induced** across the current-carrying bar.

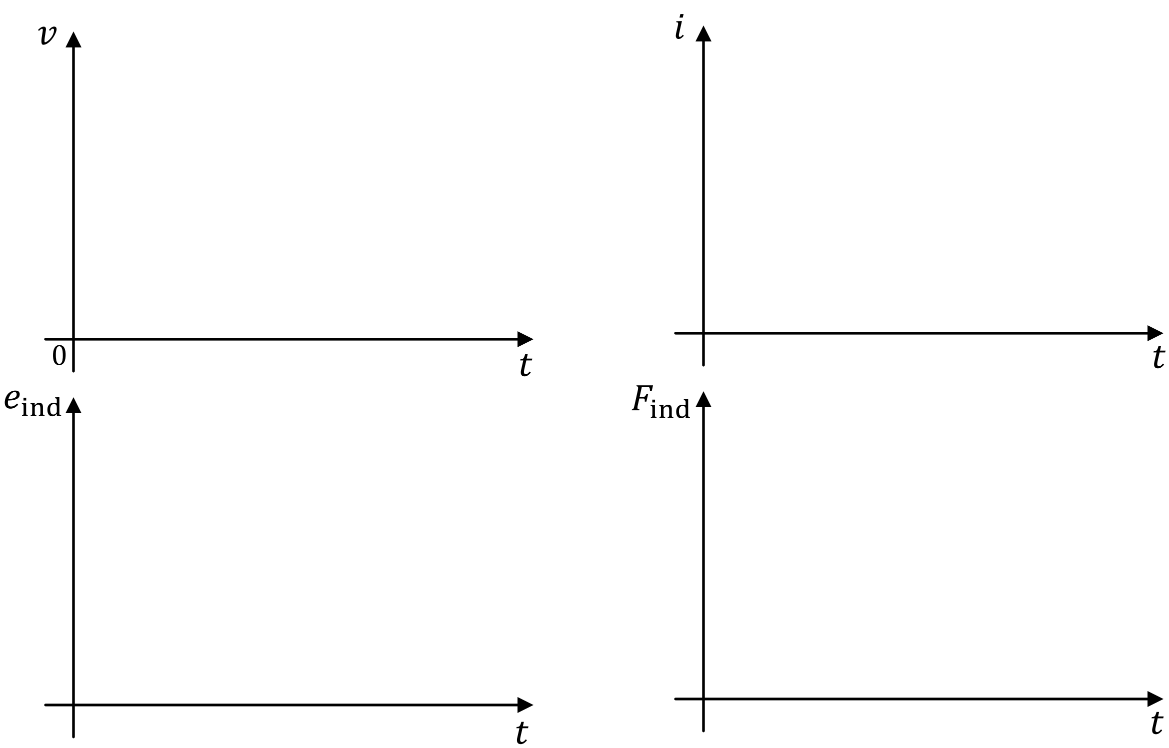
Direction of induced voltage:

4. The induced voltage will cause the current flowing to be **reduced**. Look back to Kirchhoff's voltage low:

5. This reduction in current will be followed by a **decrease** in the force production since

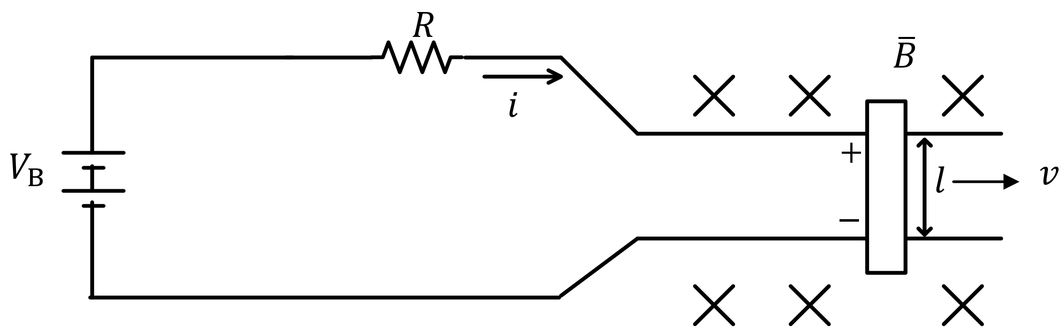
Eventually, $|\bar{F}| = 0$. At which point: $e_{\text{ind}} = V_{\text{B}}$, $i = 0$

And the bar will move at a constant **no-load speed**, $v_{\text{ss}} = \frac{V_{\text{B}}}{Bl}$



Case 2: The Linear DC machine as a motor

Assume the linear DC machine is **running at no-load** and under **steady state** conditions, i.e. steady state velocity of v_{ss}.



1. When a load force is applied in the opposite direction of motion, a net force in the bar opposite to the direction of motion exist.
2. The negative force slows down the bar, resulting in a reduction of induced voltage.
3. Following the reduction in induce voltage, the current flow in the bar increases, and the force induced or acting on the bar increases to the right side.

This force will increase until it is **equal** in **magnitude** but **opposite** in **direction to the load force**, i.e.$|\bar{F}_{\text{ind}}| = |\bar{F}_{\text{load}}|$which will **occur at a lower speed** *v*.

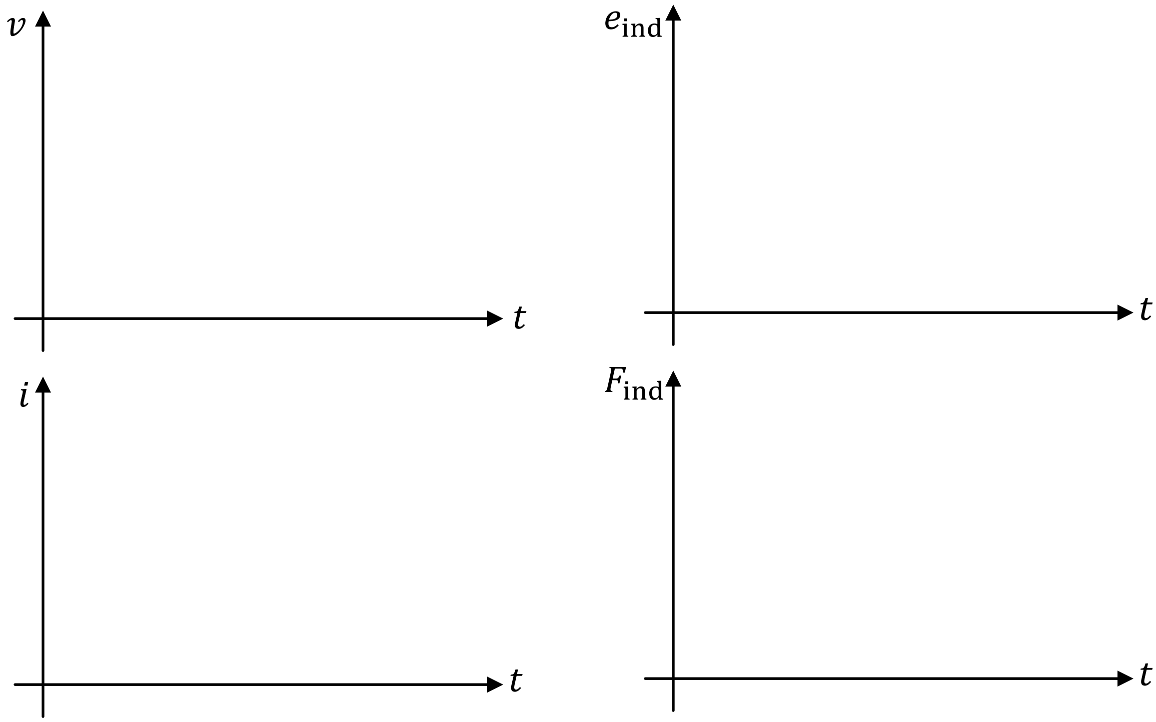
The force *F* induced in the bar is in the direction of motion of the bar and **power has been converted from electrical form to mechanical form** to keep the bar moving.

The converted power is:

$$e_{\text{ind}} i = F_{\text{ind}} v$$

Electrical power consumed ⇒ Mechanical power created

The bar is **operating as a motor** because **power** is **converted from electrical to mechanical form**.



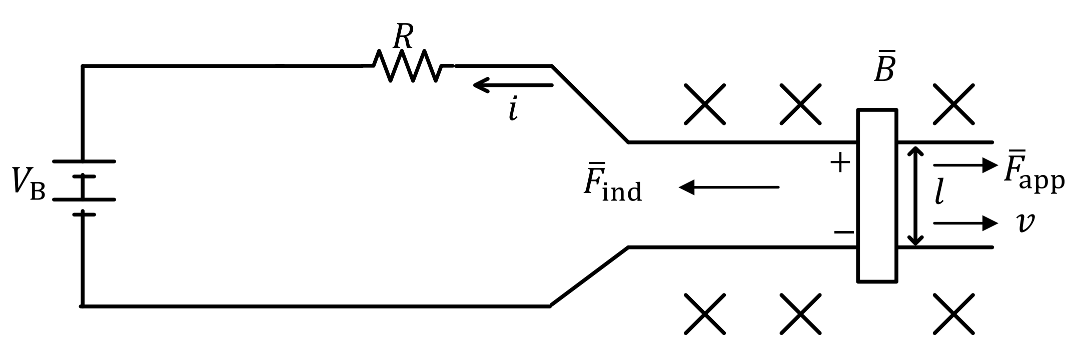
The power converter from electrical form to mechanical form in a real rotating motor is given by

$$\boldsymbol{P_{\text{conv}} = \tau_{\text{ind}}\omega}$$

Where the torque induced is the rotational analog for the induced force, and the angular velocity is the rotational analog of the linear velocity.

Case 3: The Linear DC machine as a generator

The DC machine is assumed to operating under no-load steady state conditions.



1. Force is applied in the direction of motion. Bar will accelerate in the same direction of motion.
2. Positive net force causes the bar to speed up.
3. Increase in bar speed results in the induce voltage to be greater than the battery voltage.
4. Hence current increases up the bar and the force induced increases to the left.

This will continue until $\bar{F}_{\text{ind}} = \bar{F}_{\text{app}}$ which will cause the bar to reach a new steady state and move at a higher speed *v*.
The **reversal of current** means that the linear DC **machine is now charging the battery**, i.e. it is acting as a **generator** that **converts mechanical power into electric power**.

The amount of power converted is:

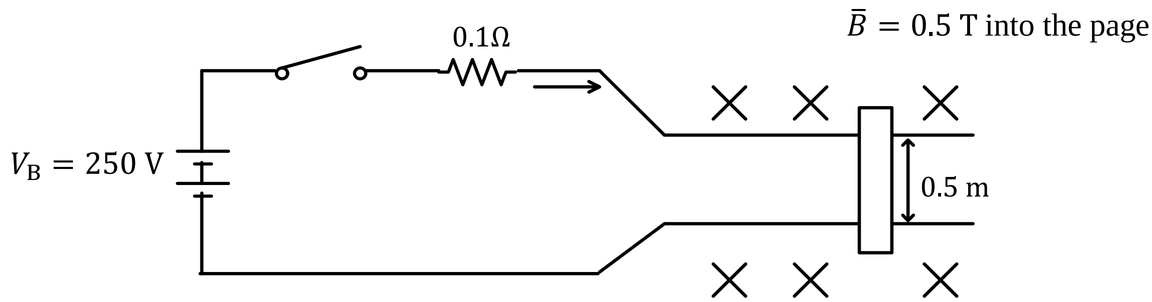
$$F_{\text{ind}} v = e_{\text{ind}} i$$

Mechanical power consumed ($F_{\text{ind}} v$) — Electrical power created ($e_{\text{ind}} i$)

Note:

- Same machine can act as **both** motor and generator.
- Difference lies in the **direction** of external force applied with respect to direction of motion.
  - Applied force opposite to direction of motion - motor
  - Applied force in the direction of motion -generator
- In both operations, induced voltage and force are both present at all times.
  - $V_{\text{B}} > e_{\text{ind}}$ - motor
  - $e_{\text{ind}} > V_{\text{B}}$ - generator
- Machine movement is **always** in the **same direction**.
  - A motor moves more slowly
  - A generator moves more rapidly

Starting problems with the Linear DC machine

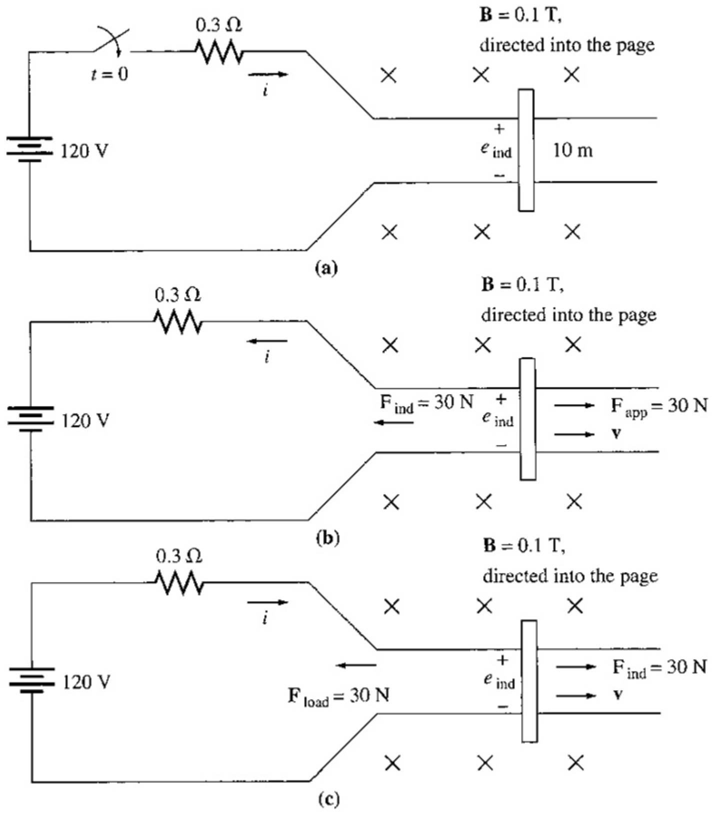


As stated previously, when the linear machine is started, there is no induced emf present, i.e. $e_{\text{ind}} = 0$. Therefore, the starting current is given by:
Typical machines have **small R** and are supplied with **rated V**, therefore the ***i*** $\boldsymbol{i_{start}}$ will be **very high** (more than 10 times rated current).

Consequence: Possibility of severe damage to motors.

Solution: **insert an extra resistance** into the circuit during starting of motor.

Example



The linear dc machine is as shown in (a).

(a) What is the machine's maximum starting current? What is the steady state velocity at no load?

(b) Suppose a 30N force pointing to the right were applied to the bar (figure b). What would the steady-state speed be? How much power would the bar be producing or consuming? Is the machine acting as a motor or a generator?

(c) Now suppose a 30N force pointing to the left were applied to the bar (figure c). What would the new steady-state speed be? Is the machine a motor or generator now?

(d) Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. What is the new speed of the bar?